Batch : Repeaters
Date : 04.03.2006

Max. Marks : 200
Duration : 2 hrs.

COMPREHENSION — 1

Complex Number

Let the vertices of a triangle A, B, C are represented by complex numbers z_1, z_2, z_3 where $|z_i| = 1$ for $i = 1, 2, 3$. If the triangle has some speciality then there will be some pure reltion among z_1, z_2, z_3 free from modulus and argument symbols. For example, if the tringle is equilateral then

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

Select the correct alternative :

1. If the tringle ABC is isosceles with AB = AC then

(A) $z_1^2 = z_2z_3$ (B) $z_2^2 = z_1z_3$ (C) $z_3^2 = z_1z_2$ (D) None of these

2. If the triangle ABC is equilateral then which of the following will not be true ?

(A) $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$ (B) $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$
(C) $z_1 + z_2 + z_3 \neq 0$ (D) $z_1z_2 + z_2z_3 + z_3z_1 = 0$

3. The orthocentre of the triangle ABC must be represented by the complex number

(A) $\sqrt{z_1^2 + z_2^2 + z_3^2}$ (B) $\sqrt{z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1}$

(C) $z_1 + z_2 + z_3$ (D) $\frac{z_1 + z_2 + z_3}{3}$

4. The area of the triangle ABC must be

(A) $\frac{i}{2}\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$ (B) $\frac{i}{4}\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$ (C) $i\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$ (D) None of these

5. If perpendicular AD from A on BC is produced to a point E on the circle $|z| = 1$ then E must have affix

(A) $-\frac{z_1z_2}{z_3}$ (B) $-\frac{z_1z_3}{z_2}$ (C) $-\frac{z_2z_3}{z_1}$ (D) $\frac{z_2z_3}{z_1}$

COMPREHENSION — 2

Theory of Equation

The location of roots of a polynomial equations with real coefficients can easily be tested by calculus. If f(x) = 0 be a polynomial equation of odd degree then it must have at least one real root. Since $f(-\infty) = -\infty$, $f(\infty) = \infty$ and f is continuous. Again if f(x) = 0 is a polynomial equation of degree 3 and f'(x) = 0 has real roots c_1 and c_2 and if $f(c_1)\,f(c_2) < 0$ then f(x) = 0 must have three real roots. A fourth degree polynomial equation may not have a root but its derived equation (i.e., f'(x) = 0) will have at least one real root.

Select the correct alternative :

6. The cubic equation $x^3 - 3x - 1 = 0$ has

(A) no real root (B) one real root (C) two real roots (D) three real roots

7. The equation $x^3 - 3x + k = 0$ will have three real roots if

(A) $0 < k < 3$ (B) $-2 < k < 2$ (C) $-3 < k < 3$ (D) $k > 3$

8. The equation $x^4 - 4x + k = 0$ has no real root if

(A) $k < 4$ (B) $k > 3$ (C) $k < 3$ (D) None of these

9. The fourth degree equation $x^4 + rx + s = 0$ will have no real roots if

(A) $27r^4 < 64s^3$ (B) $r^4 < s^3$ (C) $r^4 > s^3$ (D) $27r^4 < 256s^3$

10. If $x = \frac{3+5i}{2}$ is a root of the equation $2x^3 + ax^2 + bx + 68 = 0$ (a, b $\in$ R) then which of the following is also a root ?

(A) $\frac{5+3i}{2}$ (B) -8 (C) -4 (D) 0

[Assertion-Reasoning]

Q. No. 11 – 17 Assertion (A)–Reasoning (R) type questions

Indicate the correct choice codes for each set of (A) and (R) on the basis of following informatioin.

(A) Both ‘Assertion’ and ‘Reason’ are true and ‘Reason’ is the correct explanation of ‘Assertion’.

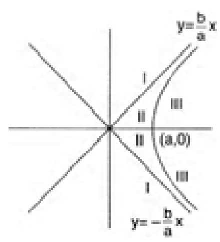
(B) Both ‘Assertion’ and ‘Reason’ are true and ‘Reason’ is not the correct explanation of ‘Assertion’.

(C) ‘Assertion’ is true but ‘Reason’ is false.

(D) ‘Assertion’ is false but ‘Reason’ is true.

(E) Both ‘Assertion’ and ‘Reason’ are false.

	Assertion		***Reasoning***
11.	If a, b, c, b, p, q, r $\in$ R and $ax^2 + bx + c \geq 0$, $px^2 + qx + r \geq 0$ for all x then $apx^2 + bqx + cr \geq 0$ for all real x.	**11.**	$ax^2 + bx + c > 0$ for all x if $a > 0$, $b^2 - 4ac < 0$.
12.	$t^3 + 3t + 3$ is a factor of $(t + 1)^{n+1} + (t + 2)^{2n-1}$ for all integral values of n.	**12.**	The roots of the equation $x^2 + 3x + 3 = 0$ are not real.
13.	If three positive numbers in G.P. represent sides of a traingle then the common ratio of the G.P. must lie between $\frac{\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}+1}{2}$.	**13.**	Three positive real numbers can form a triangle if sum of any two is greater than the third.
14.	If $D(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ where f_1, f_2, f_3 are differentiable functions and $a_2, b_2, c_2, a_3, b_3, c_3$ are constants then $\int D(x)\,dx = \begin{vmatrix} \int f_1(x)\,dx & \int f_2(x)\,dx & \int f_3(x)\,dx \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + c$	**14.**	Integration of sum of several functions is equal to sum of integration of individual function.
15.	If $a > 0$ and $b^2 - 4ac < 0$ then the value of the integral $\int \frac{dx}{ax^2 + bx + c}$ will be of the type $\mu \tan^{-1}\frac{x+A}{B} + C$, where A, B, C, μ are constants.	**15.**	If $a > 0$, $b^2 - 4ac < 0$ then $ax^2 + bx + c$ can be written as sum of two squares.
16.	In any triangle $a \cos A + b \cos B + c \cos C \leq s$.	**16.**	In any triangle $\sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{c}{2} \leq \frac{1}{8}$.
17.	If n is an odd prime then integral part of $(\sqrt{5} + 2)^n - 2^{n+1}$, ([x]) is divisible by 20 n.	**17.**	If n is prime then ${}^nC_1, {}^nC_2, {}^nC_3, \ldots {}^nC_{n-1}$ must be divisible by n.



Multiple Choice Questions

[Choose all the correct options from question 18 to 32]
[There is no negative marking]

18. If A = $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$, then

(A) $A^3 = I$ (B) $A^{-1} = A^2$ (C) $A^n = A \;\forall\; n \neq 4$ (D) All above

19. A throws n + 1 coins and B throws n coins. Let P(m, k) be the probability that A throws m heads and B throws x heads where $0 \le m \le n + 1$, $0 \le k \le n$ then

(A) $P(m, k) = \left(\frac{1}{2}\right)^{2n+1}$ (B) $P(m, k) = {}^{n+1}C_m \, {}^{n}C_k \left(\frac{1}{2}\right)^{2n+1}$

(C) $\sum_{0<k<m\le n}\sum P(m, k) = \frac{1}{2}$ (D) $\sum_{0<k<m\le n}\sum P(m, k) = \left(\frac{1}{2}\right)^{n+1}$

20. Suppose that $F(n + 1) = \frac{2F(n)+1}{2}$ for n = 1, 2, 3, ... and F(1) = 2. Then F(101)

(A) > 50 (B) is 52 (C) 54 (D) 60

21. Let $E = \frac{a^2+b^2+c^2}{ab+bc+ac}$ (where a, b, c $\in$ R, ab + bc + ac $\neq$ 0), then

(A) $E \ge 1$ for all a, b, c (B) $E \ge 1$ if ab + bc + ac > 0
(C) $E \le -2$ for all a, b, c (D) $E \le -2$ if ab + bc + ac < 0

22. Let $f(x) = x^2 + 2(m - 1)x + m + 5 = 0$ where m is a real parameter then

(A) f(x) = 0 has both roots positive if m $\in$ (– 5, – 1) (B) f(x) > 0 for all x if m $\in$ (– 1, 4)
(C) f(x) > 0 for all x > 0 if m $\in$ (– 1, ∞) (D) All above

23. If z = x + iy, then the equation $\left|\frac{2z-i}{z+1}\right| = m$ represents a circle when m =

(A) 1/2 (B) 1 (C) 2 (D) 3

Space for rough work

24. Let the expression $(7 + 4\sqrt{3})^n$ for positive integers n have integral part I and fractional part f and

(A) (I + f) (1 – f) = 1 (B) (I + f) (1 – f) = I (C) $1 - f = (7 - 4\sqrt{3})^n$ (D) f is irrational

25. Which of the following are true ?

(A) $5^{2n} + 1$ is divisible by 13 if n is even (B) $5^{2n} - 1$ is divisible by 13 if n is odd
(C) $5^{2n} - 1$ is divisible by 13 if n is even (D) $5^{2n} + 1$ is divisible by 13 if n is odd

26. The number of ways of selecting 3 pairs from 8 distinct objects

(A) 420 (B) 105 (C) 21 (D) $({}^8C_2 \cdot {}^6C_2 \cdot {}^4C_2)/3!$

27. There are n married couples at a party. Each person shakes hand with every person other than her or his spouse. The total number of hand shakes must be

(A) ${}^{2n}C_2 - n$ (B) ${}^{2n}C_2 - (n - 1)$ (C) 2n(n – 1) (D) ${}^{2n}C_2$

28. Let S_1, S_2, ... be squares such that for each $n \ge 1$, the length of a side of S_n equals to the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following value(s) of n is the area of S_n less than 1 sq. cm.?

(A) 7 (B) 8 (C) 9 (D) 10

29. If α, β, γ are cube root of p < 0, then for any x, y, z, $\dfrac{\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2}{\beta^2x^2 + \gamma^2y^2 + \alpha^2z^2}$ is

(A) 1 (B) $\dfrac{\alpha}{\gamma}$ (C) $\dfrac{\beta}{\alpha}$ (D) $\dfrac{\gamma}{\beta}$

30. If $P = n(n^2 - 1^2)(n^2 - 2^2)(n^2 - 3^2)\ .....\ (n^2 - r^2)$, n > r, $n \in N$, then P is divisible by

(A) (2r + 2)! (B) (2r – 1)! (C) (2r + 1)! (D) None of these

31. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and let $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i}, \vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a} \le 12$ then the number of values of $\vec{r}$ is

(A) ${}^{12}C_9 - 1$ (B) ${}^{12}C_3$ (C) ${}^{12}C_9$ (D) None of these

32. The number of distinct terms in the expansion of $(x_1 + x_2 + x_3 + ... + x_n)^m$ is

(A) ${}^{n+1}C_m$ (B) ${}^{n+m-1}C_m$ (C) ${}^{n+m}C_m$ (D) ${}^{n+m-1}C_{n-1}$

Space for rough work

Single Correct Questions

[Choose most appropriate alternate]

[There are negative marking for question 33 to 60]

33. If $|z-4+3i| \le 1$ and m and n be the least and greatest values of $|z|$ and k be the least value of $\left(\dfrac{x^4+x^2+4}{x}\right)$ on the interval $(0, \infty)$, then k is equal to

(A) n (B) m (C) m + n (D) None of these

34. Let z be a complex number satisfying $|z-5i| \le 1$ such that amp (z) is minimum. Then z =

(A) $\dfrac{2\sqrt{6}}{5}+\dfrac{24i}{5}$ (B) $\dfrac{24}{5}+\dfrac{2\sqrt{6}i}{5}$ (C) $\dfrac{2\sqrt{6}}{5}-\dfrac{24i}{5}$ (D) None of these

35. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors and $\Delta = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$, then

(A) $\Delta = 0$ (B) $\Delta = 1$

(C) Δ = any non-zero value (D) None of these

36. The value of $\sum_{p=1}^{6} 2\left(\sin\dfrac{2p\pi}{7} - i\cos\dfrac{2p\pi}{7}\right)$ is

(A) 2i (B) – 2i (C) 2 (D) 1

37. If the equation $e^{||x|-2|+b} = 2$ has four solutions then b lies in

(A) (ln 2 – 2, ln 2) (B) (– 2, ln 2) (C) (0, ln 2) (D) None of these

38. If $\sin\alpha$ and $\cos\alpha$ are roots of the equation $px^2 + qx + r = 0$, then

(A) $p^2 - q^2 + 2pr = 0$ (B) $(p + r)^2 = q^2 - r^2$ (C) $p^2 + q^2 - 2pr = 0$ (D) $(p - r)^2 = q^2 + r^2$

39. The equation $||x-2| + a| = 4$ can have four distinct real solutions for x if a belongs to the interval

(A) $(-\infty, -4)$ (B) $(-\infty, 0]$ (C) $[4, \infty)$ (D) None of these

Space for rough work

40. The set of values of a for which all the roots of $2^{\sin x} + a \cdot 2^{-\sin x} - 2 = 0$ are real and distinct $\left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$ is

(A) $\left[\frac{3}{4}, 1\right)$ (B) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (C) $(-8, 1)$ (D) None of these

41. If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$ then x, y, z are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

42. If $x^{18} = y^{21} = z^{28}$, then 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

43. The value of $(\ln i) + (\ln i)^2 + (\ln i)^3 + ... + (\ln i)^n$ is

(A) $\frac{n(n+1)}{2}\ln i$

(B) $\frac{\ln i}{\ln\left(\frac{i}{e}\right)}\left\{(n-1)\ln i + \ln\left(\frac{i}{e}\right)\right\}$

(C) $\frac{(i\pi)(i^n\pi^n - 2^n)}{2^n(i\pi - 2)}$

(D) $\frac{(2^n - i^n\pi^n)(2\pi i - \pi^2)}{2^n(\pi^2 + 4)}$

44. The value of $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi}$ is

(A) greater than 2 (B) less than 2 (C) equal to 0.5 (D) equal to 0

45. If y = $a^{\frac{1}{1-\log_a x}}$ and z = $a^{\frac{1}{1-\log_a y}}$, then x is equal to

(A) $a^{\frac{1}{1+\log_a z}}$ (B) $a^{\frac{1}{2+\log_a z}}$ (C) $a^{\frac{1}{1-\log_a z}}$ (D) None of these

46. The number of ways of distributing 40 different games among 8 children in such a way that three of them get 4 each, 2 of them get 5 each and remaining 3 get 6 each, is equal to

(A) [redacted] (B) $\frac{(40!).(8!)}{(4!)^3(5!)^2(6!)^3}$ (C) $\frac{(40!)}{(4!)^3(5!)^2(6!)^3}$ (D) $\frac{(8!)}{(3!)^2(2!)}$

47. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}\;\; 2\vec{b}-\vec{c}\;\; 2\vec{c}-\vec{a}\right]$ =

(A) 0 (B) 1 (C) - $\sqrt{3}$ (D) $\sqrt{3}$

Space for rough work

48. The number of different matrices that can be formed with elements 0, 1, 2 or 3 ; each matrix having 4 elements, is

(A) 3×2^4 (B) 2×4^4 (C) 3×4^4 (D) None of these

49. If a, b, c are digits, then the rational number represented by 0 . cababa ba ... is

(A) $\frac{cab}{990}$ (B) $\frac{99c+ab}{990}$ (C) $\frac{99c+10a+b}{99}$ (D) $\frac{99c+10a+b}{990}$

50. $\sum_{r=1}^{n}\left(\sum_{m=0}^{r-1} {}^nC_r \cdot {}^rC_m \cdot 2^m\right)$ is equal to

(A) $4^n - 3^n + 1$ (B) $4^n - 3^n - 1$ (C) $4^n - 3^n + 2$ (D) $4^n - 3^n$

51. If 1, α, α^2, ..., α^{n-1} are the nth roots of unity, then $\sum_{k=1}^{n-1}\frac{1}{2-\alpha^k}$ is equal to

(A) $(n-2)2^n$ (B) $\frac{(n-2)2^{n-1}+1}{2^n-1}$ (C) $\frac{(n-2)2^{n-1}}{2^n-1}$ (D) None of the above

52. If n is a positive integer and k is a positive integer not exceeding n, then $\sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2 =$

(A) $\frac{n(n+1)(n+2)}{12}$ (B) $\frac{n(n+1)^2(n+2)}{12}$ (C) $\frac{n(n+1)^2(n+2)}{6}$ (D) None of these

53. If the coefficient of x^{100} in $1 + (1 + x) + (1 + x)^2 + (1 + x)^3 + ... + (1 + x)^n$; where, $n \geq 100$, is ${}^{201}C_{101}$, then n will be equal to

(A) 100 (B) 200 (C) 101 (D) None of these

54. An urn contains a white and b black balls and another urn contains c white and d black balls. A ball is transferred from the first urn to second and then a ball is drawn from the second urn. The probability that the ball drawn from the second urn is black is

(A) $\frac{ab+bd}{(a+b)(c+d)}$ (B) $\frac{ac+bd}{(a+b)(c+d+1)}$ (C) $\frac{ad+b(d+1)}{(a+b)(c+d+1)}$ (D) $\frac{ac+bc+a}{(a+b)(c+d+1)}$

Space for rough work

55. Out of 3n consecutive integers, three are selected at random. The probability that sum of the three selected integers is divisible by 3 is

(A) $\frac{3n^2+3n+2}{(3n-1)(3n-2)}$ (B) $\frac{3n^2+3n-2}{(3n-1)(3n-2)}$ (C) $\frac{3n^2-3n+2}{(3n-1)(3n-2)}$ (D) $\frac{3n^2-3n-2}{(3n-1)(3n-2)}$

56. The outcome of an experiment are represented by points in the square bounded by x = 0 ; x = 2 ; y = 0 and y = 2 in the XOY plane. Assuming that the points are distributed uniformly, the probability that $x^2 + y^2 > 1$ is

(A) $1-\frac{\pi}{4}$ (B) $2-\frac{\pi}{4}$ (C) $1-\frac{\pi}{16}$ (D) None of these

57. The probability that event A occurs is $\frac{3}{4}$; the probability that event B occurs is $\frac{2}{3}$. Let p be the probability that both A and B occur. The smallest interval necessarily containing p is the interval

(A) $\left[\frac{1}{12},\frac{1}{2}\right]$ (B) $\left[\frac{5}{12},\frac{1}{2}\right]$ (C) $\left[\frac{1}{2},\frac{2}{3}\right]$ (D) $\left[\frac{5}{12},\frac{2}{3}\right]$

58. If $0 \le [x] < 2$; $-1 \le [y] < 1$ and $1 \le [z] < 3$, ([ ] denotes the greatest integer function) then the maximum value of determinant $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

(A) 2 (B) 6 (C) 4 (D) None of these

59. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj(adj A)) is

(A) $(14)^4$ (B) $(14)^3$ (C) $(14)^2$ (D) $(14)^1$

60. Let three digit numbers be A28, 3B9 and 62C, where A, B, C are digits between 0 and 9 and divisible by certain constant k, then determinant $\begin{vmatrix} A & 8 & 2 \\ 3 & 9 & B \\ 6 & C & 2 \end{vmatrix}$ is divisible by

(A) k (B) k^2 (C) – k (D) – k^2

Space for rough work

ANSWERS

Comprehension	Ass. & Reas.	Multiple Correct	Single Correct	
1. a	11. d	18. a,b	33. a	47. a
2. c	12. b	19. b,c	34. a	48. c
3. c	13. a	20. a,b	35. c	49. d
4. b	14. a	21. b,d	36. a	50. d
5. c	15. a	22. a,b,c	37. a	51. b
6. d	16. b	23. a,b,d	38. a	52. b
7. b	17. a	24. a,c,d	39. a	53. b
8. b		25. c,d	40. d	54. c
9. d		26. a,d	41. c	55. c
10. c		27. a,c	42. a	56. c
		28. b,c,d	43. c	57. d
		29. b,c,d	44. a	58. c
		30. b,c	45. c	59. a
		31. b,c	46. a	60. a
		32. b,d		