insight
Gateway to IITs

Batch : Repeaters | Max. Marks : 200
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MATHS

COMPREHENSION — 1

Application of Integration

Let A be fixed point on the curve y = f(x) and let S denotes the length of the arc of the curve included between A and a variable point P(x, y). Then S must be a function of x. It can be show that

$$\frac{dS}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ for some point B on } y = f(x)$$

$$\Rightarrow \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \frac{dS}{dx}\, dx$$

= (value of S when x = b) – value of s when (x = a)

= Arc AB – 0 = Arc AB.

Select the correct alternative :

1. **The length of the curve $y = \sqrt{a^2 - x^2}$ from x = 0 to x = a must be equal to**

(A) $\frac{a\pi}{2}$ (B) $3a\pi$ (C) $a\pi$ (D) None of these

2. **The arc length of the curve given by x = f"(t) cos t + f'(t) sin t ; y = – f"(t) sin t + f'(t) cos t corresponding to the interval (t_1, t_2) is given by**

(A) $f'(t_2) - f'(t_1)$ (B) $f''(t_2) - f''(t_1)$
(C) $f''(t_2) - f''(t_1) + f(t_2) - f(t_1)$ (D) $f''(t_2) - f''(t_1) + f'(t_2) - f(t_1)$.

3. **The arc length of the curve given by $x = \int_1^t \frac{\cos z}{z} dz$, $y = \int_1^t \frac{\sin z}{z} dz$ between the origin and the nearest point having a vertical tangent must be**

(A) $\log \pi$ (B) $\log(2\pi)$ (C) $\log \frac{\pi}{2}$ (D) 2π

4. **If 0 < b < a the length of the circumference of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the integral**

(A) $4a \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 t}\, dt$ (B) $4a \int_0^{\pi/2} \sqrt{1 - e \sin t}\, dt$

(C) $4a \int_0^{\pi/2} \sqrt{1 - e^2 \tan^2 t}\, dt$ (D) 4a

5. **The length of the arc of the curve y = log sec x from x = 0 to x = π/3 must be**

(A) $\log_e(1 + \sqrt{3})$ (B) $2\log(1 + \sqrt{3})$ (C) $\log_e(2 + \sqrt{3})$ (D) log 2

COMPREHENSION – 2

Application of Integration

A differential equation of the form $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = 0$ is called a linear differential equation where a_1 and a_2 are functions of x only. Inc ase a_1 and a_2 are constants, the solution of the linear differential equation can be easily written by noting following facts

(i) y = 0 is a solution of the differential equation.

(ii) if y = f(x) is a solution then y = c f(x) is also a solution.

(iii) if $y = f_1(x)$ and $y = f_2(x)$ are two solutions then $y = f_1(x) + f_2(x)$ will also be a solution.

(iv) if the distinct roots of the quadratic equation $m^2 + a_1m + a_2 = 0$ are m_1 and m_2 (real or imaginary) then the solution of the differential is $y = c_1e^{m_1x} + c_2e^{m_2x}$.

In case the roots are complex the solution can be transformed to te form $e^{ax}(c_1 \cos bx + c_2 \sin bx)$ by using Euler's theorem.

(v) In case the roots of the equation $m^2 + a_1m + a_2 = 0$ are equal (say m_1) the differential equation can be make linear by putting $\frac{dy}{dx} - m_1y = V$.

The linear differential equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 = f(x)$ can also be satisfied by some other functions which are not of the above type. **Such functions are called particular integrals.**

Select the correct alternative :

6. $y = e^x$ is a solution of the differential equation $\frac{d^2y}{dx^2} - y = 0$ then which of the following is not a solution

(A) e^{-x} (B) $ae^x + be^{-x}$ (C) $e^x + c$ (D) None of these

7. Which of the following is not a solution of $\frac{d^2y}{dx^2} - y = \sin x$?

(A) $e^x - \frac{1}{2}\sin x$ (B) $e^{-x} - \frac{1}{2}\sin x$ (C) $ae^x + be^{-x} - \frac{1}{2}\sin x$ (D) $ae^x + be^{-x} + c - \frac{1}{2}\sin x$

8. Which of the following is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

(A) $c_1 + c_2x$ (B) $(c_1 + c_2x)e^{3x}$ (C) $c_1 \cos 3x + c_2 \sin 3x$ (D) None of these

9. A particular integral of the equation $\frac{d^2y}{dx^2} + a^2y = \sin ax$

(A) $C_1 \cos ax + C_2 \sin ax$ (B) $\frac{x\sin ax}{2a^2} + \frac{\cos ax}{2a^2}$ (C) $\frac{x\sin ax}{2a^2} - \frac{\cos ax}{2a^2}$ (D) $-\frac{x\sin ax}{2a^2} - \frac{\cos ax}{2a^2}$

10. Let $D \equiv \frac{d}{dx}$ be an operator and let f(D) be a function of D, we define $\frac{1}{f(D)}X = Y$ if (f(D) Y = X. Then $\frac{1}{D-3}(x^2)$ must be equal to

(A) $-\frac{1}{3}x^2 - \frac{1}{9}x + \frac{2}{27}$ (B) $-\frac{1}{3}x^2 - \frac{2}{9}x - \frac{2}{27}$ (C) $-\frac{1}{3}x^2 + \frac{2}{9}x - \frac{2}{27}$ (D) None of these

[Assertion-Reasoning]

Q. No. 11 – 17 Assertion (A)–Reasoning (R) type questions

Indicate the correct choice codes for each set of (A) and (R) on the basis of following informatioin.

(A) Both 'Assertion' and 'Reason' are true and 'Reason' is the correct explanation of 'Assertion'.

(B) Both 'Assertion' and 'Reason' are true and 'Reason' is not the correct explanation of 'Assertion'.

(C) 'Assertion' is true but 'Reason' is false.

(D) 'Assertion' is false but 'Reason' is true.

(E) Both 'Assertion' and 'Reason' are false.

Assertion	Reasoning
11. The domain of the function $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is $[-1, 1]$.	**11.** $\sin^{-1} x$, $\cos^{-1} x$ are defined for $\lvert x \rvert \le 1$ and $\tan^{-1} x$ is defined for all x.
12. If $f(x + y) = f(x) + f(y)$, then f is either differentiable everywhere or not differentiable everywhere.	**12.** Any function is either differentiable everywhere or not differentiable everywhere.
13. The equation $x^2 = x \sin x + \cos x$ has only one solution.	**13.** The derivative of the function $x^2 - x \sin x - \cos x$ is $x(2 - \cos x)$.
14. The function $\frac{\sin(x+a)}{\sin(x+b)}$ $(a \ne b)$ has at least one extreme value between a and b.	**14.** An equation of the type $\frac{1}{f(x)} = 0$ has no solution.
15. If n is a positive integer then $\int_0^{n\pi} \left\lvert \frac{\sin x}{x} \right\rvert dx \ge \frac{2}{n}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right)$.	**15.** Over $\left(0,\frac{\pi}{2}\right)$, $\frac{\sin x}{x} \ge \frac{2}{\pi}$.
16. The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is least if $k = 0$.	**16.** The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is $\sqrt{k^2+20}$.
17. The differential equation $y^3 dy + (x + y^2)dx = 0$ becomes homogeneous if we put $y^2 = t$.	**17.** All differential equation of first order and first degree becomes homogeneous if we put $y = tx$.

Multiple Choice Questions

[Choose all the correct options from question 18 to 35]
[There is no negative marking]

18. Which of the following function is periodic (where, [x] denotes the greatest integer function)

(A) $\text{sgn}(e^{-x})$ (B) $\sin x + |\sin x|$ (C) $\min\{\sin x, |x|\}$ (D) $\left[x+\frac{1}{2}\right]+\left[x-\frac{1}{2}\right]+2[-x]$

19. Let $f(x + y) + f(x - y) = 2f(x)\, f(y)\ \forall\, x, y \in$ □ and $f(0) = k$, then

(A) f is even if k = 1 (B) f is odd if k = 0
(C) f is always odd (D) f is neither even nor odd for any k

20. If $f(x) = |x - 1| - [x]$, where [x] is the greatest integer less than or equal to x, then

(A) $f(1 + 0) = -1,\ f(1 - 0) = 0$ (B) $f(1 + 0) = 0 = f(1 - 0)$
(C) $\lim_{x\to 1} f(x)$ exists (D) $\lim_{x\to 1} f(x)$ does not exist

21. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0 & x \in \square \\ x^2 & x \in \square - \square \end{cases}$. Then

(A) $\lim_{x\to 1} g(x)$ exists, but g(x) is not continuous at x = 1
(B) $\lim_{x\to 1} f(x)$ does not exist and f(x) is not continuous at x = 1
(C) gof is continuous for all x (D) fog is continuous for all x

22. The function $f(x) = \max\{(1 - x), (1 + x), 2\}$, $x \in (-\infty, \infty)$ is

(A) continuous at all points (B) differentiable at all points
(C) differentiable at all points except at x = 1 and x = - 1
(D) continuous at all points except at x = 1 and x = - 1, where it is discontinuous

23. Let $f(x) = (ax + b)\cos x + (cx + d)\sin x$ and $f'(x) = x \cos x$ be an identity in x, then

(A) a = 0 (B) b = 1 (C) c = 1 (D) d = 0

24. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in$ □ and $f_0(x) = x$ then $\frac{d}{dx}\{f_n(x)\}$ is equal to

(A) $f_n(x).\frac{d}{dx}\{f_{n-1}(x)\}$ (B) $f_n(x).f_{n-1}(x)$
(C) $f_n(x).f_{n-1}(x). \ldots .f_2(x).f_1(x)$ (D) None of these

Space for rough work

25. A particle is moving in a straight line such that its distance at any time t is $S = \frac{t^4}{4} - 2t^3 + 4t^2 + 7$, then

(A) velocity is max at $t = \frac{(6-2\sqrt{3})}{3}$ (B) acceleration is min at t = 2

(C) the distance is min at t = 0, 4 (D) None of these

26. Let $f(x) = ax^3 + bx^2 + cx + 1$ have extrema at $x = \alpha, \beta$ such that $\alpha\beta < 0$ and $f(\alpha).f(\beta) < 0$ then the equation f(x) = 0 has

(A) three equal real roots (B) three distinct real roots

(C) one positive root if $f(\alpha) < 0$ & $f(\beta) > 0$ (D) one negative root if $f(\alpha) > 0$ & $f(\beta) < 0$

27. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve $x = a\sin^3 t$, $y = a\cos^3 t$ at an arbitrary point, then

(A) $4OT^2 + ON^2 = a^2$ (B) the length of the tangent = $\left|\frac{y}{\cos t}\right|$

(C) the length of the normal = $\left|\frac{y}{\sin t}\right|$ (D) None of these

28. If $I = \int_0^1 \sqrt{1+x^3}\, dx$ then

(A) $I < 1$ (B) $I \neq \frac{\sqrt{5}}{2}$ (C) $I < \frac{\sqrt{7}}{2}$ (D) None of these

29. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then

(A) $I_7 + I_5 = \frac{1}{6}$ (B) $I_{10} + I_8 = \frac{1}{9}$ (C) $I_8 - I_{12} = \frac{2}{99}$ (D) $I_{12} + 2I_{10} + I_8 = \frac{20}{99}$

30. The solution of $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ is

(A) $\sqrt{x^2+y^2} = a\left[\sin\left(\tan^{-1}\frac{y}{x}\right) + c\right]$ (B) $\sqrt{x^2+y^2} = a\left[\cos\left(\tan^{-1}\frac{y}{x}\right) + c\right]$

(C) $\sqrt{x^2+y^2} = a\left[\tan\left(\sin^{-1}\frac{y}{x}\right) + c\right]$ (D) $y = x\tan\left(c + \sin^{-1}\frac{1}{a}\sqrt{x^2+y^2}\right)$

Space for rough work

31. The solution of $y = x\left[\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3\right]$ are given by $\left(\text{where, } p = \frac{dy}{dx}\right)$

(A) The constant function y = 0

(B) $y = kp^{-3}e^{-\frac{p^2}{2}}(p + p^3)$

(C) $y = p^3e^{-\frac{p^2}{2}}(p + p^3)$

(D) $y\,e^{-\frac{p^2}{2}} = p^{-2} + 1$

32. The solution of $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\left(\frac{dy}{dx}\right) = y^2$ is

(A) $y - \frac{c}{1+\cos x} = 0$

(B) $y = \frac{c}{1-\cos x}$

(C) $x = 2\sin^{-1}\sqrt{\frac{c}{2y}}$

(D) None of the above

33. If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions f(0) = - 1, f'(log 2) = 31, $\int_0^{\log 4}(f(x)-cx)\,dx = \frac{39}{2}$, then

(A) a = 5

(B) b = - 6

(C) c = 2

(D) a = 3

34. Let f(x) = cos x sin 2x then

(A) $\min_{x\in(-\pi,\pi)} f(x) > -\frac{7}{9}$

(B) $\min_{x\in(-\pi,\pi)} f(x) > -\frac{9}{7}$

(C) $\min_{x\in[-\pi,\pi]} f(x) > -\frac{1}{9}$

(D) $\min_{x\in[-\pi,\pi]} f(x) > -\frac{2}{9}$

35. If - 1, $f(x) = \begin{vmatrix}\sec\theta & \tan^2\theta & 1\\ \theta\sec x & \tan x & x\\ 1 & \tan x-\tan\theta & 0\end{vmatrix}$ and $6^n\sin\left(2x+\frac{n\pi}{2}\right)\cos\left(\frac{3x+n\pi}{2}\right)$ are in AP for all x, y and y_n, then

(A) 0

(B) $y = \int_0^x f(t)\sin\{k(x-t)\}dt$

(C) $y = \sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\,\sin\beta\,\cos(\alpha+\beta)$

(D) $\frac{1}{\sqrt{1+9y^2}}$

Space for rough work

Single Correct Questions

[Choose most appropriate alternate]

[There are negative marking for question 36 to 60]

36. If f(x), g(x) be twice differentiable function on [0, 2] satisfying f”(x) = g”(x), f’(1) = 2, g’(1) = 4 and f(2) = 3, g(2) = 9, then f(x) - g(x) at x = 4 equals

(A) 0 (B) - 10 (C) 8 (D) 2

37. If $\int g(x)\{f(x) + f'(x)\}dx = f(x)\, g(x)$, then | g(x) | is

(A) $e^{-x}e^{c}$ (B) $e^{x}e^{c}$ (c) $\ln e^{x}e^{c}$ (D) $-\ln e^{-x}e^{c}$

38. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by

(A) $x^n + n^2y$ = constant (B) $ny^2 + x^2$ = constant (C) $n^2x + y^n$ = constant (D) $n^2x - y^n$ = constnat

39. The area enclosed by the curve $|y| = \sin 2x,\ 0 \le x \le 2\pi$ is

(A) 1 (B) 2 (C) 3 (D) 4

40. Let f(x) = MIN $\left\{\tan x, \cot x, \frac{1}{\sqrt{3}}\right\}$ $\forall\ x \in \left[0, \frac{\pi}{2}\right]$. The area bounded by y = f(x) and the x-axis is

(A) $\ln\left(\frac{4}{3}\right) + \frac{\pi}{6\sqrt{3}}$ (B) $\ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\pi}{12\sqrt{3}}$ (C) $\ln\left(\frac{4}{3}\right) + \frac{\pi}{12\sqrt{3}}$ (D) $\ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\pi}{6\sqrt{3}}$

41. If the area bounded by the curve y = f(x), (f(x) > 0, $\forall\ x \in \mathbb{R}^+$) the x-axis, the y-axis and the line x = a if $\frac{6a - \sin 2a}{4}$ $\forall$ a > 0, then f(x) is

(A) $1 + \sin^2 x,\ x > 0$ (B) $1 - \cos^2 x,\ x > 0$ (C) $\frac{1+\cos^2 x}{4},\ x > 0$ (D) $\frac{1-\cos^2 x}{4},\ x > 0$

42. Area bounded by the curve y = $\sqrt{\sin[x] + [\sin x]}$, (where, [ ] is the greatest integer function), lines x = 1 and x = $\frac{\pi}{2}$ and the x-axis is

(A) $\left(\frac{\pi}{2} - 1\right)$ (B) $\sqrt{\frac{\pi}{2}}\left(\frac{\pi}{2} - 1\right)$ (C) $\sqrt{\sin 1}\left(\frac{\pi}{2} - 1\right)$ (D) $\sqrt{\cos 1}\left(\frac{\pi}{2} - 1\right)$

Space for rough work

43. If $f(x) = \int_0^x \frac{1}{\{f(t)\}^2} dt$ and $\int_0^2 \frac{1}{\{f(t)\}^2} dt = \sqrt[3]{6}$, then f(9) is equal to

(A) 2 (B) 0 (C) 3 (D) None of these

44. If A(x), B(x), C(x) are quadratic expressions, $\Delta(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A'(x) & B'(x) & C'(x) \\ A''(x) & B''(x) & C''(x) \end{vmatrix}$ and $\Delta(2) = 5$, then $\int_{-3}^{3} \Delta(x)\, dx$ is equal to

(A) 0 (B) 10 (C) 20 (D) 30

45. $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}} dx$ is equal to

(A) $\frac{1+\sqrt{x}}{(1-x^2)}$ + c (B) $\frac{1+\sqrt{x}}{(1+x)^2}$ + c (C) $\frac{1-\sqrt{x}}{(1-x)^2}$ + c (D) $\frac{2(\sqrt{x}-1)}{\sqrt{1-x}}$ + c

46. $\int \frac{e^{x^2}(2x+x^3)}{(3+x^2)^2} dx$ is equal to

(A) $\frac{e^{x^2}}{(3+x^2)}$ + c (B) $\frac{1}{2}\frac{e^{x^2}}{(3+x^2)^2}$ + c (C) $\frac{1}{4}\frac{e^{x^2}}{(3+x^2)^2}$ + c (D) $\frac{1}{2}\frac{e^{x^2}}{(3+x^2)}$ + c

47. $f(x) = \int \cos^2(\ln x)\, dx$ is equal to

(A) $\frac{x}{2} + \frac{x\cos(2\ln x) + 2x\sin(2\ln x)}{10} + c$ (B) $\frac{x}{10}$ [5 + cos(2 ln x) - 2 sin (2 ln x)] + c

(C) $\frac{x}{10}$ [5 + cos(2 ln x) + 2 sin(2 ln x)] + c (D) None of the above

48. If $f(x) = (ab - b^2 - 2)x + \int_0^x (\cos^4\theta + \sin^4\theta)\, d\theta$ is a decreasing function of x for all x ∈ □ and b ∈ □, b being independent of x, then

(A) $a \in (0, \sqrt{6})$ (B) $a \in (-\sqrt{6}, \sqrt{6})$ (C) $a \in (-\sqrt{6}, 0)$ (D) None of these

Space for rough work

49. If l, m are the direction cosines of a ray in a plane then maximum vlaue of lm is

(A) 1/4 (B) 1/2 (C) 1 (D) None of these

50. If $(f(x))^n = f(nx)$, then $\frac{f'(nx)}{f'(x)}$ =

(A) $\frac{f(x)}{f(nx)}$ (B) $\frac{f(nx)}{f(x)}$ (C) f(nx) . f(x) (D) None of these

51. If F(x) = $\frac{1}{x^2}\int_4^x \left(4t^2 - 2F'(t)\right)$ dt, then F'(4) equals

(A) $\frac{32}{9}$ (B) $\frac{64}{3}$ (C) $\frac{64}{9}$ (D) None of these

52. If f'(x) = sin x + sin 4x . cos x then f' $\left(2x^2 + \frac{\pi}{2}\right)$ at x = $\sqrt{\frac{\pi}{2}}$ is equal to

(A) 0 (B) – 1 (C) – 2 $\sqrt{2\pi}$ (D) None of these

53. If f : $\mathbb{R} \to \mathbb{R}$ and f(x) = | |x| - 1| is not differentiable at

(A) 0, ± 1 (B) ± 1 (C) 0 (D) 1

54. Let f(x) = $\begin{cases} \sqrt{1+x^2} & x < \sqrt{3} \\ \sqrt{3}x - 1 & \sqrt{3} \le x < 4 \\ [x] & 4 \le x < 5 \\ |1-x| & x \ge 5 \end{cases}$; where [x] is the greatest integer less than or equal to x. The number of points of discontinuity of f(x) in $\mathbb{R}$ is

(A) 3 (B) 0 (C) infinite (D) None of these

Space for rough work

55. If f(x) = min {tan x, cot x}, then

(A) f(x) is not differentiable at x = 0, $\frac{\pi}{4}, \frac{5\pi}{4}$ (B) f(x) is discontinuous at x = 0, $\frac{\pi}{2}, \frac{3\pi}{2}$

(C) $\int_0^{\pi/2} f(x)\,dx = \ln 2$ (D) All of these

56. The number of points at which the function f(x) = $\frac{1}{\log|x|}$ is discontinuous is

(A) 1 (B) 2 (C) 3 (D) 4

57. Let us consider f(x) = 1 - x + [x] - [1 - x] and g(x) = 1 - x - $\lim_{n\to\infty}$ $(\cos^{2n+1} \pi x)$

(A) The graph of f(x) is a straight line passing thorugh origin
The graph of g(x) is a straight line passing thorugh origin
(B) The graph of both f(x) and g(x) is a straight line not passing through origin
(C) The graph of f(x) is a straight line passing through origin whereas for that of g(x) is not passing through origin
(D) will not be a straight line for either of the curve

58. $\lim_{n\to\infty}\left\{\frac{7}{10}+\frac{29}{10^2}+\frac{133}{10^3}+\dots+\frac{5^n+2^n}{10^n}\right\}$ is equal to

(A) $\frac{3}{4}$ (B) 2 (C) $\frac{5}{4}$ (D) $\frac{1}{2}$

59. If $\lim_{n\to\infty}\frac{n^k \sin^2 n!}{n+1}$ = 0, where n is any integer for

(A) k $\in$ □ (B) k < 1 (C) k = 1 (D) k > 1

60. $\lim_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$ (where, [ ] denotes the greatest integer function)

(A) equal to 1 (B) equal to 0 (C) Does not exist (D) None of these

Space for rough work

ANSWERS

Comprehension	Ass. & Reas.	Multiple Correct	Single Correct	
1. a	11. a	18. a,b,c,d	33. a,b	47. a
2. c	12. c	19. a,b	34. a,b	48. b
3. c	13. d	20. a,d	35. a,c	49. b
4. a	14. d	21. a,b,c	36. b	50. b
5. c	15. b	22. a,c	37. b	51. a
6. c	16. c	23. a,b,c,d	38. b	52. c
7. d	17. c	24. a,c	39. d	53. a
8. b		25. a,b,c	40. a	54. d
9. d		26. b,c,d	41. a	55. d
10. b		27. a,b,c	42. c	56. c
		28. a,c	43. c	57. d
		29. a,b,c,d	44. d	58. c
		30. a,d	45. d	59. b
		31. a,b	46. d	60. b
		32. a,b,c		