3

Revision Test

insight
Gateway to IITs

Batch : Repeaters	**Max. Marks : 200**
Date : 14.03.2006	**Duration : 2 hrs.**

MATHS

COMPREHENSION — 1

Conics

If we rotate axes through an angle 45° in the clockwise direction, the equation of the rectangular hyperbola $x^2 - y^2 = a^2$ reduces to $xy = \frac{a^2}{2}$ to $xy = c^2$ (writing c^2 for $\frac{a^2}{2}$). This hyperbola is easier to handle. Any point on this hyperbola may be taken as $\left(ct, \frac{c}{t}\right)$.

Select the correct alternative :

1. The asymptotes to the hyperbola $xy = c^2$ must be

(A) $y = x, y = -x$ (B) $y = cx, y = -cx$ (C) $y = 0, x = 0$ (D) None of these

2. The equation of chord joining $\left(ct_1, \frac{c}{t_1}\right)$ and $\left(ct_2, \frac{c}{t_2}\right)$ on $xy = c^2$ is

(A) $x + yt_1t_2 = c(t_1 + t_2)$ (B) $y + xt_1t_2 = c(t_1 + t_2)$ (C) $y + x(t_1 + t_2) = ct_1t_2$ (D) None of these

3. Equation of normal at $\left(ct, \frac{c}{t}\right)$ must be

(A) $t^2x - t^2y - ct^3 + c = 0$ (B) $t^2x - t^2y - ct^3 + c = 0$ (C) $t^3x - ty = c$ (D) $t^3x - ty - ct^4 + c = 0$

4. If the normal to the rectangular hyperbola $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ meets the curve again at $\left(ct_1, \frac{c}{t_1}\right)$ then t_1 must be equal to

(A) $-\frac{1}{t}$ (B) $-\frac{1}{t_2}$ (C) $-\frac{1}{t_3}$ (D) $-t$

5. The line $y = mx + 2c\sqrt{-m}$ touches the hyperbola $xy = c^2$ at

(A) $\left(\frac{c}{\sqrt{-m}}, c\sqrt{-m}\right)$ (B) $\left(\frac{c}{\sqrt{-2m}}, c\sqrt{-2m}\right)$ (C) $\left(\frac{c}{\sqrt{m}}, c\sqrt{m}\right)$ (D) None of these

COMPREHENSION – 2

Circles

The equation of a circle passing through the vertices of a triangel ABC can be found by elementary methods. If the equation of sides are given to us, we can find the equation of circumcircle of Δ ABC by writing a second degree curve through A, B, C and then choose particular values of constants to make it a circle. Indeed if equation of BC, CA and AB of triangle ABC are $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ then a second degree curve through A, B, C may be taken as
$\lambda L_2L_3 + \mu L_3L_1 + \nu L_1L_2 = 0$. Note that this is a second degree curve passing through A, B and C. If this represents a circle then coeff. of x^2 = coeff. of y^2 and coeff. of $xy = 0$.

Select the correct alternative :

6. The equation $\lambda L_2L_3 + \mu L_3L_1 = 0$ is also satisfied by points A, B, C. What locus does it represent ?

(A) circle (B) parabola (C) ellipse (D) pair of lines

7. $\lambda L_2^2L_3 + \mu L_3^2L_1 + \nu L_1^2L_2 = 0$ **is also satisfied by A, B, C. What locus does it represent ?**
(A) circle (B) parabola (C) an ellipse (D) None of these

8. **The equation of the circumcircle of the triangle when the equation of whose sides are $y = x$, $y = 2x$, $y = 3x + 2$ is**
(A) $x^2 + y^2 - 6x - 8y = 0$ (B) $x^2 + y^2 + 6x - 8y = 0$ (C) $x^2 + y^2 - 6x + 8y = 0$ (D) None of these

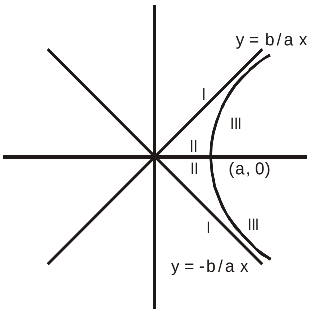
9. **The value of k for which the circumcircle of the triangle the equation of whose sides are, $y = x$, $y = 2x + 1$ and $y = 3x + k$ passes through origin is**
(A) – 1 (B) 1 (C) 0 (D) 2

10. **$L_1 = 0$, $L_2 = 0$ be two distinct parallel lines. $L_3 = 0$, $L_4 = 0$ be two other distinct parallel lines which are not parallel to L_1 or L_2. Then they will form a parallelogram. The equation of circle passing through the vertices of the parallelogram must be of the form**
(A) $\lambda L_1L_4 + \mu L_2L_3 = 0$ (B) $\lambda L_1L_3 + \mu L_2L_4 = 0$ (C) $\lambda L_1L_2 + \mu L_3L_4 = 0$ (D) None of these

[Assertion-Reasoning]

Q. No. 11 – 17 Assertion (A)–Reasoning (R) type questions

Indicate the correct choice codes for each set of (A) and (R) on the basis of following informatioin.
(A) Both ‘Assertion’ and ‘Reason’ are true and ‘Reason’ is the correct explanation of ‘Assertion’.
(B) Both ‘Assertion’ and ‘Reason’ are true and ‘Reason’ is not the correct explanation of ‘Assertion’.
(C) ‘Assertion’ is true but ‘Reason’ is false. **(D)** ‘Assertion’ is false but ‘Reason’ is true.
(E) Both ‘Assertion’ and ‘Reason’ are false.

Assertion	*Reasoning*
11. The lines $(a + b)x + (a - 2b)y = a$ are concurrent at the point $\left(\frac{1}{3}, \frac{2}{3}\right)$.	**11.** The lines $x + y - 1 = 0$ and $x - 2y = 0$ intersect at the point $\left(\frac{1}{3}, \frac{2}{3}\right)$.
12. The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$	**12.** The area of the triangle formed by three concurrent lines must be zero.
13. If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then $f'g = fg'$.	**13.** Two circles touch each other, if line joining their centres is perpendicular to all possible common tangents.
14. The equation of chord of the circle $x^2 + y^2 - 6x + 10y - 9 = 0$ which is bisected at $(-2, 4)$ must be $x + y - 2 = 0$.	**14.** In notations the equation of the chord of the circle $S = 0$ bisected at (x_1, y_1) must be $T = S_1$.
15. If a point (x_1, y_1) lie in the region II of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ shown in the figure then $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} < 0$. 	**15.** If a point $P(x_1, y_1)$ lies outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} < 1$.
16. The equation $x^2 + 2y^2 + \lambda xy + 2x + 3y + 1 = 0$ can never represent a hyperbola.	**16.** The general equation of second degree represent a hyperbola if $h^2 > ab$.
17. The line $y = \frac{b}{a}x$ will not meet the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > b > 0)$.	**17.** The line $y = \frac{b}{a}x$ is an asymptotes to the hyperbola.

Multiple Choice Questions

[There is no negative marking] [Choose all the correct options from question 18 to 32]

18. ABC is an isosceles triangle whose base is BC. If B and C are (a + b, b – a) and (a – b, a + b), then co–ordinates of A may be

(A) (a, b) (B) (b, a) (C) $\left(\frac{a}{b}, \frac{b}{a}\right)$ (D) $\left(1, \frac{b}{a}\right)$

19. If the lines $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form three sides of a square, the equation of the fourth side may be

(A) $2x - y - 6 = 0$ (B) $2x - y + 6 = 0$ (C) $2x - y - 14 = 0$ (D) $2x - y + 14 = 0$

20. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (– 1, 2) satisfy

(A) $3x + 2y \geq 0$ (B) $2x + y - 13 \geq 0$ (C) $2x - 3y - 12 \leq 0$ (D) $-2x + y \geq 0$

21. The lines $x + y - 1 = 0$, $(m - 1)x + (m^2 - 7)y - 5 = 0$ and $(m - 2)x + (2m - 5)y = 0$ are

(A) concurrent for three values of m (B) concurrent for one value of m
(C) concurrent for no value of m (D) are parallel for m = 3

22. Two adjacent vertices of a square are (2, – 1) and (– 1, 3) then the other vertices may be

(A) (– 5, 0) (B) (– 2, – 4) (C) (3, 6) (D) (6, 2)

23. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through the point (– 2, 11) is

(A) $4x + 3y = 25$ (B) $7x - 24y = 320$ (C) $3x + 4y = 38$ (D) $24x - 7y + 125 = 0$

24. The point $(\lambda, 1 + \lambda)$ lies inside the circle $x^2 + y^2 = 1$ for

(A) $\lambda = -½$ (B) $\lambda < 0$ (C) $-1 < \lambda < 0$ (D) All λ

25. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touches each other, then α may be

(A) 0 (B) 1 (C) – 4/3 (D) 4/3

26. If OP and OQ are the tangents from (0, 0) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ then

(A) Equation of PQ is $gx + fy = 0$ (B) Equation of PQ is $gx + fy + c = 0$
(C) Equation of circle OPQ is $x^2 + y^2 + gx + fy = 0$ (D) Equation of circle OPQ is $x^2 + y^2 + 2gx + 2fy = 0$

27. $C_1 : x^2 + y^2 = 25$, $C_2 : x^2 + y^2 - 2x - 4y - 7 = 0$ be two circles intersecting at A and B

(A) Equation of common chord of C_1 and C_2 must be $x + 2y - 9 = 0$
(B) Eqution of common chord must be $x + 2y + 7 = 0$
(C) Tangents at A and B to the circle C_1 intersect at $\left(\frac{25}{9}, \frac{50}{9}\right)$
(D) Tangents at A and B to the circle C_1 intersect at (1, 2).

28. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$, $r > 1$ represents

(A) an ellipse (B) a hyperbola (C) a circle (D) None of these

29. If $\frac{x}{ma} + \frac{y}{nb} = 1$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

(A) $m^2 = \frac{n^2}{n^2 - 1}$ (B) $n^2 = \frac{m^2}{m^2 - 1}$ (C) $m^2 = \frac{n^2}{n^2 + 1}$ (D) $n^2 = \frac{m^2}{m^2 + 1}$

Space for rough work

30. Let P(a secθ, b tanθ) and Q(a secϕ, b tanϕ) where $\theta + \phi = \pi/2$ be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q then k is equal to

(A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

31. The equation of a sphere described on the join of the points A and B having position vectors $2\hat{i}+6\hat{j}-7\hat{k}$ and $-2\hat{i}+4\hat{j}-3\hat{k}$ respectively as a diameter is

(A) $x^2 + y^2 + z^2 - 10y + 10z + 41 = 0$ (B) $\left[\vec{r}-(2\hat{i}+6\hat{j}-7\hat{k})\right]\left[\vec{r}-(-2\hat{i}+4\hat{j}-3\hat{k})\right] = 0$

(C) $\left[(x-2)\hat{i}+(y-6)\hat{j}+(z+7)\hat{k}\right]\left[(x+2)\hat{i}+(y-4)\hat{j}+(z+3)\hat{k}\right] = 0$ (D) $x^2 + (y-5)^2 + (z+5)^2 = 9$

32. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then

(A) $d^2 + (3b - 2c)^2 = 0$ (B) $d^2 + (3b + 2c)^2 = 0$ (C) $d^2 + (2b - 3c)^2 = 0$ (D) $d^2 + (2b + 3c)^2 = 0$

Single Correct Questions

[There are negative marking for question 33 to 60]

33. If $a \neq b \neq c$ and if $ax + by + c = 0$; $bx + cy + a = 0$, $cx + ay + b = 0$ are concurrent then $2^{a^2b^{-1}c^{-1}}\, 2^{b^2c^{-1}a^{-1}}\, 2^{c^2a^{-1}b^{-1}} =$

(A) 8 (B) 0 (C) 2 (D) None of these

34. Let the perpendiculars from any point on the line $7x + 56y = 0$ upon $3x + 4y = 0$ and $5x - 12y = 0$ be p and p', then

(A) 2p = p' (B) p = 2p' (C) p = p' (D) None of these

35. If f(x + y) = f(x) f(y) for all x and y, f(1) = 2, then area enclosed by $3|x| + 2|y| \leq 8$ is

(A) f(5) sq. unit (B) f(6) sq. unit (C) $\frac{1}{3}$ f(6) sq. unit (D) f(4) sq. unit

36. If one of the lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between positive directions of the axes, then a, b, h satisfy the relation

(A) $a + b = 3|h|$ (B) $(a + b)^2 = 4h^2$ (C) $a - b = 2|h|$ (D) $(a - b)^2 = 4h^2$

37. If p, q > 0 and p < q and $px^2 + 4\mu xy + qy^2 + 4a(x + y + 1) = 0$ represents a pair of straight lines, then

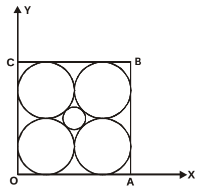
(A) $p \leq a \leq q$ (B) $a \leq p$ (C) $a \geq p$ (D) $a \leq p$ or $a \geq q$

38. If lines given by equation $ax^2 - bxy - y^2 = 0$ make angle α, β with x–axis then, $\tan(\alpha+\beta) =$

(A) $\frac{b}{1+a}$ (B) $-\frac{b}{1+a}$ (C) $\frac{a}{1+b}$ (D) $-\frac{a}{1+b}$

Space for rough work

39. In the figure OABC is a square of side 8 cm, then the equation of the smallest circle is



(A) $(x-4)^2 + (y-4)^2 = 4$
(B) $(x-4)^2 + (y-4)^2 = 8$
(C) $(x-4)^2 + (y-4)^2 = 12$
(D) None of these

40. A circle of radius 5 unit touches both the axes and lies in the first quadrant. If the circle makes one complete roll on x–axis along the positive direction x–axis, then its equation in the new position is
(A) $x^2 + y^2 + 20\pi x - 10y + 100\pi^2 = 0$
(B) $x^2 + y^2 + 20\pi x + 10y + 100\pi^2 = 0$
(C) $x^2 + y^2 - 20\pi x - 10y + 100\pi^2 = 0$
(D) None of these

41. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is
(A) a parabola
(B) a circle
(C) an ellipse
(D) a pair of straight lines

42. Four distinct points $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ lying on $xy = 1$ are concyclic if
(A) $abcd = 1$
(B) $abcd + 1 = 0$
(C) $ab = cd$
(D) $ac = bd$

43. Equation of the circle through the intersection of $x^2 + y^2 + 2x = 0$ and $x - y = 0$, having minimum radius is
(A) $x^2 + y^2 - 1 = 0$
(B) $x^2 + y^2 - x - y = 0$
(C) $x^2 + y^2 - 2x - 2y = 0$
(D) None of these

44. If $x + y = k$ is normal to $y^2 = 12x$, then k is
(A) 3
(B) 9
(C) – 9
(D) – 3

45. If the tangents at P and Q on a parabola meet in T, then SP, ST and SQ are in
(A) A.P.
(B) G.P.
(C) H.P.
(D) None of these

46. If the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ cuts the parabola again at $Q(aT^2, 2aT)$ then
(A) $-2 \le T \le 2$
(B) $T \in (-\infty, -8) \cup (8, \infty)$
(C) $T^2 < 8$
(D) $T^2 \ge 8$

47. If the focus of the parabola is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$, then the length of latus rectum of the parabola is
(A) $\frac{u^2}{g}\cos^2\alpha$
(B) $\frac{u^2}{g}\cos 2\alpha$
(C) $\frac{2u^2}{g}\cos 2\alpha$
(D) $\frac{2u^2}{g}\cos^2\alpha$

48. If the line $y = \sqrt{3}\,x - 3$ cuts the parabola $y^2 = x + 2$ at P and Q and if A be the point $(\sqrt{3}, 0)$, then AP . AQ is
(A) $\frac{2}{3}(\sqrt{3} + 2)$
(B) $-\frac{4}{3}(\sqrt{3} + 2)$
(C) $\frac{4}{3}(2 - \sqrt{3})$
(D) $2\sqrt{3}$

Space for rough work

49. An ellipse has eccentricity 1/2 and one focus at the point P(1/2, 1). Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. The equation of the ellipse, in the standard form, is

(A) $\frac{(x-1/3)^2}{1/9} + \frac{(y-1)^2}{1/12} = 1$ (B) $\frac{(x-1/3)^2}{1/8} + \frac{(y-1)^2}{1/12} = 1$ (C) $\frac{(x-1/3)^2}{1/9} + \frac{(y-1)^2}{1/8} = 1$ (D) None

50. The locus of mid points of chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that touch the circle $x^2 + y^2 = b^2$

(A) $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^4} + \frac{y^2}{b^4}$

(B) $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = b^2\left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)$

(C) $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = a^2\left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)$

(D) None of these

51. The chord PQ of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ subtends an angle of 90° at the centre, the locus of intersection point of tangents at P and Q is

(A) a circle (B) an ellipse (C) a hyperbola (D) a line

52. If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then its eccentric angle θ is equal to

(A) 0 (B) 90° (C) 45° (D) 60°

53. If the normal at any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, meets the axes in G and g respectively, then PG : Pg =

(A) a : b (B) $a^2 : b^2$ (C) $b^2 : a^2$ (D) b : a

54. The locus of the point $\left(\frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2}\right)$ is a hyperbola of eccentricity

(A) $\sqrt{3}$ (B) 3 (C) $\sqrt{2}$ (D) 2

Space for rough work

55. If $(a \sec\theta, b \tan\theta)$ and $(a \sec\phi, b \tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\phi}{2}\right) =$

(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{e+1}{e-1}$

56. The centre of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is C. At any point P on hyperbola the tangent line is drawn, which cuts the lines $bx - ay = 0$ and $bx + ay = 0$ at Q and R, then CQ . CR =

(A) $a^2 - b^2$ (B) $a^2 + b^2$ (C) $\frac{1}{a^2}+\frac{a}{b^2}$ (D) $\frac{1}{a^2}-\frac{1}{b^2}$

57. Consider the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Area of the triangle formed by the asymptotes and the tangent drawn to it at (a, 0) is

(A) ab/2 (B) ab (C) 2ab (D) 4ab

58. If H(x, y) = 0 represent the equation of a hyperbola and A(x, y) = 0, C(x, y) = 0, the equations of its asymptotes and the conjugate hyperbola respectively, then for any point (α,β) in the plane, $H(\alpha,\beta)$, $A(\alpha,\beta)$ and $C(\alpha,\beta)$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None of these

59. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then

(A) $4al + n = 0$ (B) $4al + 4am + n = 0$ (C) $4am + n = 0$ (D) $al + n = 0$

60. The radius of director circle of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(A) a - b (B) $\sqrt{a-b}$ (C) $\sqrt{a^2-b^2}$ (D) $\sqrt{a^2+b^2}$

Space for rough work

ANSWERS

Comprehension	Ass. & Reas.	Multiple Correct	Single Correct	
1. c	11. a	18. a,d	33. a	47. d
2. a	12. d	19. b,c	34. c	48. b
3. d	13. c	20. a,c	35. c	49. a
4. c	14. d	21. c,d	36. b	50. b
5. a	15. d	22. a,b,c,d	37. d	51. b
6. d	16. d	23. a,d	38. b	52. c
7. d	17. a	24. a,c	39. d	53. c
8. c		25. c,d	40. d	54. c
9. c		26. b,c	41. b	55. b
10. c		27. a,c	42. a	56. b
		28. d	43. d	57. b
		29. a,b	44. b	58. a
		30. d	45. b	59. a
		31. a,b,c,d	46. d	60. c
		32. d		